Waves

Waves: Wave is a distrubance from equilibrium position, which travels.

It is the distance which travels, not the particles, particles only oscillates.

* Wave is a form of Energy transfer and Momentum tranfer.

Distance may be in the form of :-

i) displacement e.g waves on string
ii) Pressure e.g sound wave
iii) density
iv) Electric and magnetic field e.g electromagnetic wave (EMW)

Types of Waves

1) Mechanical wave

- Requires a medium to travel

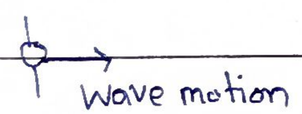
e.g Sound wave

2) Non-mechanical wave

- Do not require a medium to travel

e.g Light waves

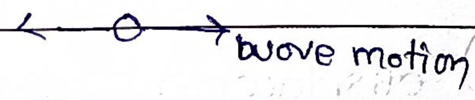
3) Transverse Wave



* Oscillation of particle ~~lar~~ perpendicular to direction of Propagation

- In Solids and liquid surface only

4) Longitudinal wave



* Oscillation of particles is in the direction of propagation of wave

* In solid liquid gas

e.g Sound waves

Plane Progressive (travelling) Harmonic Wave

Wave in which all particles are doing SHM

Equation of Travelling Harmonic wave

$$\boxed{y = A\sin(Kx \pm \omega t)}$$

y = disp. of particle from mean position

A = Amplitude

k = Wave number $\left(k = \frac{2\pi}{\lambda}\right)$

x = Wave travels along x

ω = Angular frequency

Velocity of wave $= \frac{\omega}{k} = \frac{2\pi f}{\frac{2\pi}{\lambda}} = \boxed{v = f\lambda}$

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Direction of Harmonic Wave

$$\left.\begin{matrix} \omega t + Kx \\ -\omega t - Kx \end{matrix}\right\} \text{Same sign} \rightarrow -\text{ve direction}$$

$$\left.\begin{matrix} \omega t - Kx \\ Kx - \omega t \end{matrix}\right\} \text{opposite sign} - +\text{ve direction}$$

Phase and phase difference ($\Delta\phi$)

$$y = A\sin(\underbrace{\omega t + Kx}_{\text{phase}} + \phi)$$

$$y_1 = A_1 \sin(\underbrace{K_1 x + \omega_1 t}_{\phi_1})$$

$$y_2 = A_2 \sin(\underbrace{K_2 x + \omega_2 t}_{\phi_2})$$

$$\Delta\phi = \phi_2 - \phi_1 = (K_2 x + \omega_2 t) - (K_1 x + \omega_1 t)$$

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Wave velocity and Particle velocity

$$y = A\sin(Kx - \omega t)$$

Wave velocity = $v = \dfrac{\text{Coefficient } t}{\text{Coefficient of } x} = \dfrac{-\omega}{K}$

$$\boxed{v = f\lambda}$$

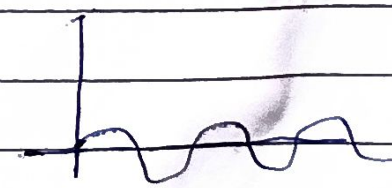
Particle velocity

y = displacement of particle

$$V_p = \frac{dy}{dt} = \boxed{V_{p\,max} = \omega A}$$

Relation b/w wave velocity and Particle velocity

$$V_p = -v\left(\frac{\partial y}{\partial x}\right)$$



Acceleration of Particle

$$y = A\sin(kx - \omega t)$$

$$V_p = \frac{dy}{dt} = -\omega A\cos(kx - \omega t)$$

$$a_p = \frac{dV_p}{dt} = -\omega^2 \underbrace{A\sin(kx - \omega t)}_{y}$$

$$\boxed{a_p = -\omega^2 y}$$

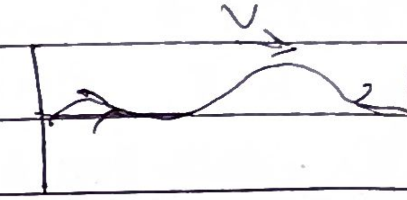
$$a_{p_{max}} = \omega^2 A$$ extreme position

a in SHM is always towards mean position

Speed of wave on string

Depends on medium

$$V = \sqrt{\frac{T}{\mu}}$$



T = tension on string

μ = mass per unit leng

$$m = \rho \times V$$

$$m = \rho(\pi r^2 l)$$

$$\frac{m}{l} = \rho \pi r^2$$

$$\mu = \rho \pi r^2$$

$$\boxed{V = \sqrt{\frac{T}{\rho \pi r^2}}}$$

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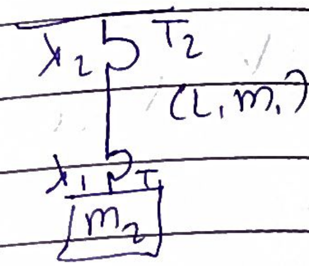
A uniform ~~s~~ rope of length l and mass m_1 hangs vertically from a rigid support

(f, u → constant)



$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{m_1+m_2}{m_2}}$$

Intensity of Wave :-

Intensity - Amount of energy passing normally through a unit area per second

$I = \frac{E}{At}$ Unit → watt/m^2

$I = 2\pi^2 f^2 A^2 \rho V$

$I \propto A^2$

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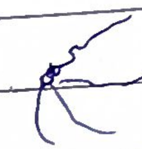
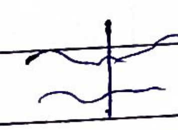
Intensity with distance from source :-

1) Point source  $I \propto \frac{1}{d^2}$

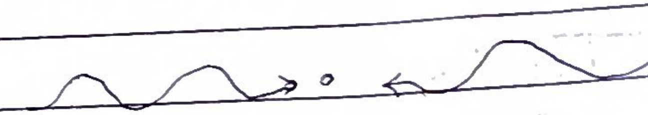
(d = distance)

2) Linear source

 $I \propto \frac{1}{d}$

Interference of waves

(two wave meets)



($\vec{y}_{net} = y_1 + y_2$ (displacement)) (Principle of superposition)

Resultant Amplitude & Intensity

$y_1 = A_1 \sin(\omega t_1 + kx_1)$
$y_2 = A_2 \sin(\omega t_2 + kx_2)$ $\Big\} + \phi$

$y_{net} = y_1 + y_2$

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A Phase difference (ϕ) $= (\omega t_2 + kx_2) - (\omega t_1 + kx_1)$

$(A_R)^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$)

$(I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

Constructive Interference	Destructive
Waves meet	Waves Seperate
$I_R, A_R \rightarrow$ maximum	$I_R, A_R \rightarrow$ minimum
$(\cos\phi) \rightarrow$ max. (1)	$(\cos\phi) \rightarrow$ minimum
$(\phi = 0, 2\pi)\cdots(2n\pi)$	$(\phi = -1), (\phi = (2n-1)\pi)$
Path difference (Δx)	Path difference (Δx)
$\Delta x = 0, \lambda, 2\lambda \ldots$	
($\Delta x = n\lambda$)	$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}$
Put $\cos\phi = 1$ in A_R	
I_R formula	$(\Delta x = (2n-1)\lambda/2$
$\cdot (A_R)_{max} = A_1 + A_2$	Put $\cos\phi = -1$
$I_{R\,max} = (\sqrt{I_1} + \sqrt{I_2})^2$	

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$\cdot (A_R)_{min} = A_1 - A_2$

$(I_R)_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

$$\boxed{\frac{I_{max}}{I_{min}} = \left(\frac{A_{max}}{A_{min}}\right)^2}$$

	Reflection (same medium)	Transmission (re... (different medi...
Velocity (v)	Same	Change
frequency (f)	Same	Same
wavelength (λ)	Same	Change
Phase difference	Rarer $\rightarrow$ Denser ($\Delta\phi = \pi$) Denser $\rightarrow$ Rarer ($\Delta\phi = 0$)	$\Delta\phi = 0$

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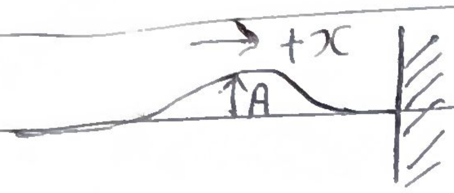
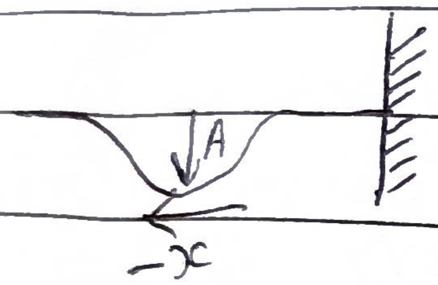
Reflaction of wave from fixed end



(No loss of energy)

$$y_i = A\sin(Kx - \omega t)$$



$$y_r = -A\sin(Kx + \omega t$$

Reflation from free end (Rarer)

$$A_R = \left(\frac{V_2 - V_1}{V_1 + V_2}\right) A_i$$

$$A_t = \left(\frac{2V_2}{V_1 + V_2}\right) A_i$$